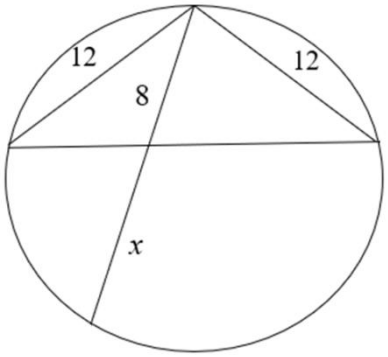
Mathematical Olympiad
Reshetnev Siberian State University of Science and Technology,
May 2025

1. Prove that $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$.

2. Find the length of a line segment *x* from the figure:



3. Solve the system of equations:

$$\begin{cases} x^2 - 2xy + 2y^2 + 2x - 8y + 10 = 0, \\ 2x^2 - 7xy + 3y^2 + 13x - 4y - 7 = 0. \end{cases}$$

4. In the UK and the USA, temperature is measured in Fahrenheit. For example, the melting point of ice is $32°F$, and the boiling point of water is $212°F$. Is there a temperature at which the Celsius and Fahrenheit values are the same?

5. It is known that one circle is inscribed in one of the faces of a unit cube, and another circle is circumscribed around the adjacent face of this cube. Find the smallest distance between the points of these circles.

6. Find the vector $\vec{x}$ if it is known that

$$(\vec{x}, \vec{a}) = 1, \ (\vec{x}, \vec{b}) = 0, \ (\vec{x}, \vec{c}) = 0, \ (\vec{a}, \vec{b}, \vec{c}) = 1.$$

7. Find the area of a figure bounded by the following lines:

$$y = x^2, \ y = 8x^2, \ x = y^2, \ x = 8y^2.$$

8. A parabola $y = -\frac{1}{2}x^2 + px + q$ intersects the abscissa axis at points *A* and *B*, a point *C* is the vertex of the parabola, and the area of a triangle *ABC* is equal to $1/2$. Find the angle between the tangentlines to this parabola at the points *A* and *B*.

9. In a swamp *A*, three candidates (*B*, *C* and *D*) were vying for the position of a conductor of the frog orchestra. According to the election rules, voting was carried out by croaking in support of the chosen candidate, and one could remain silent (keep mum) no more than once out of three times. The "croakmeter" showed that 60% of the votes of all the frogs who took part in the voting were for the candidate *B*, 70% were for the candidate *C*, and 85% were for the candidate *D*. What was the percentage of the frogs that have croaked three times?

10. If a student oversleeps the morning wake-up time, he is late with a probability of 0.9. In addition, even if this student gets up on time, he may be late due to unforeseen circumstances that occur approximately once every 10 days. In this case, the probability of being late is 0.5. If the student is late on average once every 5 days, what is his probability of oversleeping?